

CAFE Info Sheet 4

How is CAFE calculated from these tests?

Fuel economy is calculated from the emissions generated during the tests. The emissions are analyzed for their carbon content and the test fuel economy is calculated using the carbon balance equation following. We know how much carbon is in a typical gallon of test fuel and that all the carbon comes from the fuel used or the ambient air. So we can determine the fuel economy by measuring the carbon compounds expelled in the exhaust. Three carbon compounds are expelled and measured; hydrocarbons (HC), carbon monoxide (CO), and carbon dioxide (CO₂). One analysis is done for the city test and one for the highway test using the following equation (see 40 CFR 600.113-88 for full explanation of equation):

$$\text{MPG} = \frac{(5174)(10^4)(\text{CWF})(\text{SG})}{(\text{CWF}(\text{HC}) + 0.429(\text{CO}) + 0.273(\text{CO}_2)) \times ((0.6)(\text{SG})(\text{NHV}) + 5471)}$$

where HC = grams/mile HC CWF = carbon weight fraction of test fuel
CO = grams/mile CO NHV = net heating value by mass of test fuel
CO₂ = grams/mile CO₂ SG = specific gravity of test fuel

$$\text{Fecomb} = \frac{1}{\frac{.55}{\text{City FE}} + \frac{.45}{\text{hwy FE}}}$$

This formula is the equivalent of multiplying the fuel consumed per mile during the city test by 55 percent and multiplying the fuel consumed per mile during the highway test by 45 percent and adding the result. Since we do not actually measure the fuel consumed, we must perform the inverse and divide the fuel economy proportionately. This is known as a harmonic mean (sometimes called harmonic averaging).

Once the fuel economy is determined for all vehicles tested, then they are sales-weighted and the average of the ratios is determined using this equation:

$$\text{FE ave} = \frac{\text{TOTAL SALES}}{\frac{\text{sales}_1}{\text{fe}_1} + \frac{\text{sales}_2}{\text{fe}_2} + \dots + \frac{\text{sales}_n}{\text{fe}_n}}$$

where TOTAL SALES is only the sales of the tested configurations and the ratios are the sales for each tested configuration.

All the values from different configurations must be combined into a single value for each base level. Then all the values from different base levels must be combined into a single value for each model type. Model type values are rounded to a tenth of a mpg and then combined into the Corporate Average Fuel Economy value. The above equation is used to determine an average for base level, model type, and the CAFE.

The final value is adjusted to account for any changes to the test procedures since the base year (1975). All of these values are expressed as miles per gallon (mpg).

What is harmonic averaging and why is it used?

To understand why we use harmonic averaging, first understand that CAFE is trying to measure the fuel consumed or how many gallons are used when driving a mile (gal/mi). Calculating a simple arithmetic average of miles divided by gallons (mpg) yields the wrong result. The answer implicitly assumes each car uses the same number of gallons, instead of driving the same distance. For example, suppose

Car A goes 100 miles and uses 10 gallons of fuel getting 10 mpg.
Then Car A goes 100 miles and uses 5 gallons of fuel getting 20 mpg. The average fuel economy is equal to total miles driven divided by the

$$\frac{100 + 100}{10 + 5} = \frac{200}{15} = 13.33 \text{ mpg.}$$

However, the simple arithmetic average of the individual vehicle fuel economy values would be

$$\frac{10 + 20}{2} = 15 \text{ mpg.}$$

A harmonic mean (gm) of mpg averages gallons per mile, which CAFE is doing as in the first equation above. The formula for a harmonic mean is

$$UH. = \frac{N}{\sum_{i=1}^N \frac{1}{x_i}}$$

$$\frac{1}{\frac{1}{10} + \frac{1}{20}}$$

The fuel economy equations represent examples of harmonic averaging or using a harmonic mean. Computing the harmonic mean of the individual fuel economy in the above example would yield

$$\frac{2}{\frac{1}{10} + \frac{1}{20}} = 13.3 \text{ mpg; the true average fuel economy}$$

Harmonic averaging must be used to produce an accurate figure for the average fuel economy of the fleet. Calculating an arithmetic average of the individual fuel economy values does not equal the actual fuel economy for the vehicles as a group because it does not accurately account for varying fuel consumption. Using harmonic averaging allows the calculation of mpg equivalent to an arithmetic average of gallons per mile (gal/mi). Remember, CAFE actually represents an average of fuel consumption, not an average of fuel economy values.

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